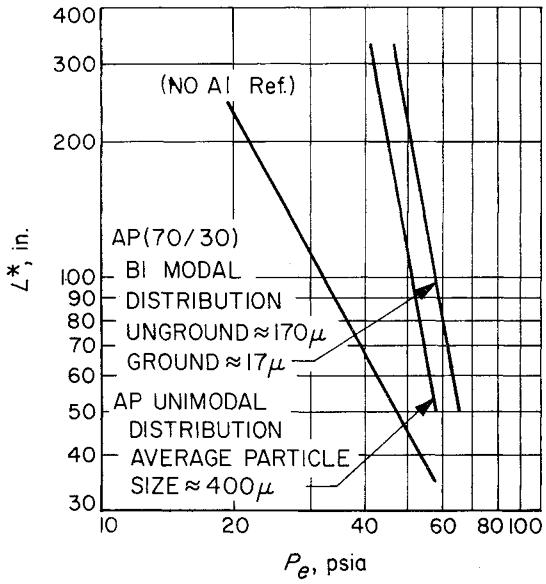

Fig. 4 Effect of AP particle size on L^* vs P_e relationship, 3-in.-diam motor, $T_0 = 80°F$, 16% Al in propellant, average Al particle size ≈ 7 μ.

igniter. Finally, satisfactory ignitions were obtained by attaching strips of a different easily ignitable propellant to these charges. The subject of sensitivity of ignition to pressure and the controlling factors for ignition with varying concentration and particle size of Al and AP in the propellant are currently under investigation. The results of the tests for the evaluation of coarser oxidizer particle size in propellant are shown in Figs. 4 and 5. The pertinent conclusion that can be drawn is that the slope of L^* vs extinction pressure relationship is not affected by the variation of oxidizer particle size in the propellant, and the variation in extinction pressure at a given L^* is small.

Some of the conclusions, which can be drawn from the various phases of the experimental program conducted at JPL can be summarized briefly as follows: 1) for a given propellant, the L^* vs extinction pressure relationship is a valid parameter for determining the low-pressure stable combustion limit; 2) for nonaluminized propellant, the slope of L^* vs P_e is

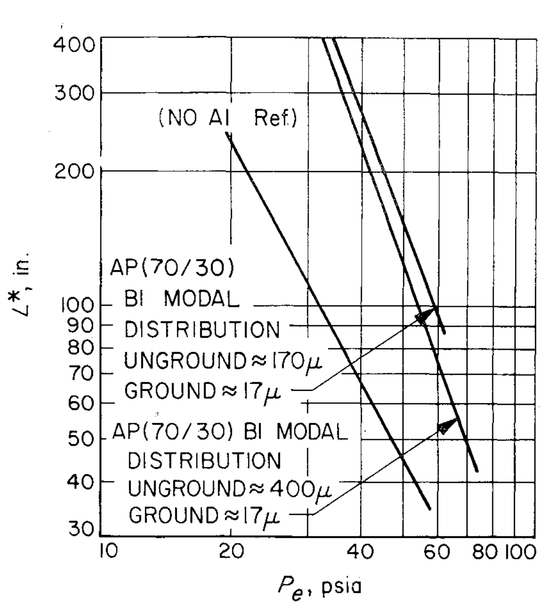

Fig. 5 Effect of AP particle size on L^* vs P_e relationship, 3-in.-diam motor, $T_0 = 80°F$, 16% Al in propellant, average Al particle size ≈ 31 μ.

$-2n$, as predicted theoretically for critical pressure; 3) the slope of L^* vs P_e relationship is affected by the presence of Al, the concentration of Al, and the Al particle size in propellant; 4) the effect of coarser Al in propellant is to cause incomplete combustion at low pressures, and the burning and extinction characteristics tend to approach those of nonaluminized propellant; and 5) the variation in oxidizer particle size has a negligible effect on the L^* vs P_e relationship; the small differences in extinction pressure at a given L^* are attributed to the variation in packing density.

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Use of Biot's Variational Technique in Heat Conduction

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RECENTLY several papers using Biot's method have appeared indicating an increased interest in the use of variational techniques in the determination of temperature profiles. Because, under some conditions, the variational techniques admit several solutions, it is well to comment on the technique.

When the temperatures are prescribed on the surfaces and polynomial representations are used for the temperature profile in the media (e.g., Refs. 1–3), one normally introduces into the polynomial certain free parameters (termed generalized coordinates) that are to be determined by the solution of Biot's differential equations. In general, for a given profile, there will be only one solution. If, however, the heat flux and not the surface temperature is prescribed, then an additional free parameter for the temperature of each surface must be included. Now all of these generalized coordinates may be determined through Biot's equations and Lardner's[3] energy content equation, or one may use the surface flux conditions and reduce the number of variational equations. These two methods produce differing results, and the choice

Table 1

$\dot{H}_x\|_0 = F$	$-k(\partial\theta/\partial x)\|_0 = F$
$H_x = Ft[1 - (x/q_2)]^3$	$H = \left(\dfrac{Fc}{6k}\right) q_2^2 \left[1 - \left(\dfrac{x}{q_2}\right)\right]^3$
$V = 9F^2t^2/10cq_2$	$V = cF^2q_2^3/40k^2$
$D = \dfrac{1}{2k}\left\{\dfrac{F^2q_2}{3} + \dfrac{3F^2t^2\dot{q}_2^2}{35q_2} + \dfrac{F^2t\dot{q}_2}{7q_2}\right\}$	$D = \dfrac{11}{24}\,\dfrac{F^2c^2q_2^3\dot{q}_2^2}{35k^3}$
$q_2 = 2.81\,(\alpha t)^{1/2}$	$q_2 = 2.65\,(\alpha t)^{1/2}$

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Table 2

$\dot{H}_x\|_0 = F$	$-k(\partial\theta/\partial x)_0 = F$
$\theta(x = 0) = 1.068(F/c)(t/\alpha)^{1/2}$ $-k(\partial\theta/\partial x)\|_0 = 0.761\ F$ Energy content $= Ft$	$\theta(x = 0) = 1.32(F/c)(t/\alpha)^{1/2}$ $-k(\partial\theta/\partial x)\|_0 = F$ Energy content $= \frac{7}{6}\ Ft$
Exact solution	
$T(x = 0) = 1.13\ (Fc)(t/\alpha)^{1/2}$ $-\ k(d\theta/dx\|_0 = F$	

of which to use depends upon the use to which the solution will be put.

These methods correspond to the collocation methods commonly employed in solid mechanics where one either satisfies the field equations exactly and the boundary conditions approximately or satisfies the boundary conditions exactly and the field equations approximately. Since the limitations of the collocation methods are well known to those in applied mechanics, but because most workers in heat conduction are unfamiliar with the differing results of the two methods, it is of value to indicate the differences.

To illustrate the point, consider a semi-infinite region that at time zero has a constant heat flux F applied to its surface. We may choose the temperature profile to be $\theta = q_1[1 - (x/q_2)]^2$ where θ is the temperature measured above the initial value, q_1 represents the surface temperature, x is the distance from the surface, and q_2 is the penetration depth. The boundary conditions are

$$-k(\partial\theta/\partial x)|_{x=0} = F \qquad \text{and} \qquad k(\partial\theta/\partial x)|_{x=q_2} = 0$$

Now the heat-flux vector $\mathbf{H}$, defined by $\text{div}\mathbf{H} = -c\theta$, reduces to the single component H_x and is given by[1]

$$H_x = (cq_1q_2/3)[1 - (x/q_2)]^3$$

The generalized coordinates are related now by either the flux boundary condition or by the total energy balance given by the relation[3]

$$\text{energy addition rate} = \int_S \frac{\partial H_x}{\partial t}|_0\, d\sigma = \dot{H}_x|_0 A = FA$$

Table 1 indicates the pertinent results. Even though the thermal potential and dissipation function are quite different, the penetration depths are in good agreement. However, when one considers the surface heat flux and total energy content one obtains results as shown in Table 2. It is seen that these values are greatly different and that either the boundary condition or the energy content are noticeably in error. Obviously, if the purpose of the calculation were to compute the energy loss in a surrounding media, or to evaluate the average stress in the region, the heat balance must be satisfied. On the other hand if the gradient at the surface is of importance, e.g., for the calculation of the gradient of thermal stresses for crack propagation, then the exact boundary condition must be used. Either case may be used if only the surface temperature is desired. Consequently one must exercise care in the method used to determine those generalized coordinates that pertain to the surface conditions.

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Modification of Magnetic Energy by Differential Fluid Motions

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I. Introduction

As the solar plasma moves up in the solar atmosphere to form solar wind, many important physical phenomena are produced by the interaction between the solar plasma and the magnetic field encountered by the solar plasma. The interplanetary magnetic field is believed to originate from the general field of the sun, for example, by the convective motion of the solar wind against the general field.[1]

A basic question is how a magnetic field is modified by the motion of an electrically conducting fluid when the kinetic energy of the flow is well above the magnetic energy. Clark[2] has studied this problem by considering a unidirectional magnetic field $H(y, t)$ acted upon by a two-dimensional stagnation flow given by

$$u = Ax \qquad v = -Ay \tag{1}$$

where x, y are Cartesian coordinates; u, v are the x and y velocity components, respectively; and A is a positive constant. He found that the flow tends to increase the magnetic energy by a stretching of the field lines and to increase the rate of dissipation through Joule heating.

The purpose of this work is to show that the same flow can act, however, to diminish the magnetic energy, if the unidirectional magnetic field is rotated 90° in the x-y plane (Fig. 1) such that

$$H_x = 0 \qquad H_y = H(x, t) \tag{2}$$

Diminution, instead of amplification, of the magnetic energy can be expected, since the flow acts to expand the lines of force.

II. Solutions and Discussions

By using Eqs. (1) and (2), the magnetohydrodynamic equations[2,3] become

$$\partial H/\partial t + A(\partial/\partial x)(xH) = \eta(\partial^2 H/\partial x^2) \tag{3}$$

where η is the magnetic diffusivity. This equation differs from the basic equation, Eq. (7), in Clark's paper by the sign in the second term of the left-hand side. It follows immediately that the total magnetic energy E and the total rate of dissipation D defined by

$$E(t) = \frac{\mu}{8\pi}\int_{-\infty}^{\infty} [H(x, t)]^2 dx$$

and

$$D(t) = \frac{\mu\eta}{4\pi}\int_{-\infty}^{\infty} \left[\frac{\partial H(x, t)}{\partial x}\right]^2 dx$$

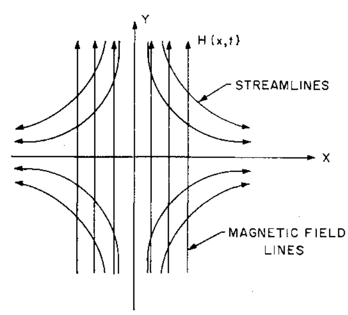

Fig. 1 Streamlines of stagnation flow and magnetic field lines.

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